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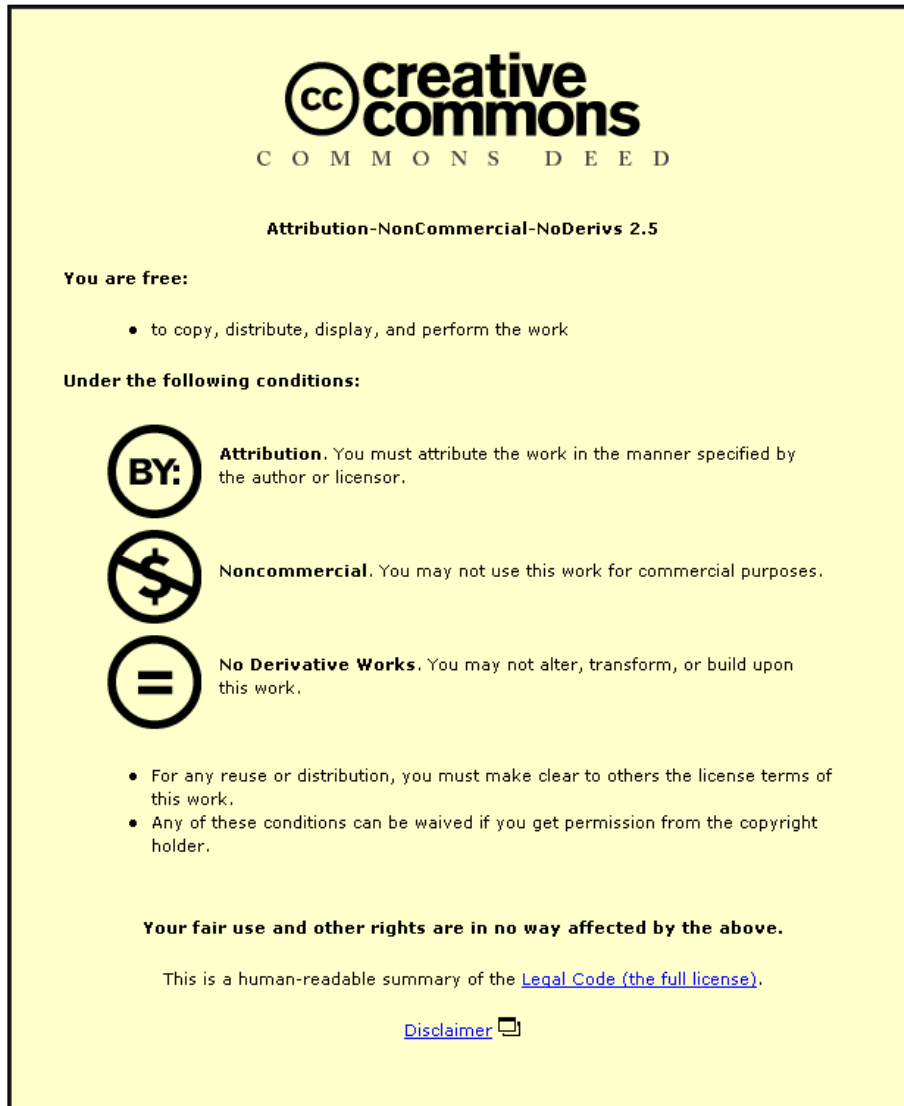
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
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## MODELLING CROSSFLOW ELECTRO- AND MICRO- FILTRATIONS

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### ABSTRACT

Experimental data from a tubular crossflow filter, operated with and without imposed d.c. electric fields, are presented. The drop-off of filtrate rate as the membrane fouls has been monitored for a variety of operating conditions. A simple mathematical model has been developed to interpret the experimental results; the model facilitates particle trajectory (and hence membrane fouling) calculations from a knowledge of the particle properties and suspension flow parameters.

### INTRODUCTION

Separation of fine (particulate) species from relatively dilute suspensions is a common requirement in the traditional production of commodity chemicals, the lower tonnage but highly specific manufacture of pharmaceuticals, and a range of fermentation processes. Microfiltration units are widely used for the clarification and sterilization of liquids, and although capable of removing particles of size down to about 0.1  $\mu\text{m}$  microfiltration is only suitable for low concentration feed streams. Ultrafiltration systems utilise a high velocity transverse flow over a polymeric membrane to prevent particle deposition or cake formation; the pores in the asymmetric membranes have sizes of the order of 0.02  $\mu\text{m}$  or smaller. The use of larger pore size media with crossflow is known as microfiltration to distinguish it from other membrane methods. Slurry thickeners utilising the crossflow concept have been available for several years<sup>1</sup> but these are generally limited to concentrating suspensions of particles of diameter larger than about 5  $\mu\text{m}$ .

Crossflow microfiltration involves the motion of a suspension tangential to the surface of a filter medium or membrane; the geometric form of the filter is either that of a 'plate and frame type filter', or 'tubular' with the filter being a porous tube mounted centrally inside a solid tube and the feed is pumped into the annular space between the two.

Whilst the bulk flow is tangential to the medium or membrane there is also a convective flow into the porous wall which causes particles to be convected laterally towards the membrane. The particle concentration near the membrane surface can increase significantly and even result in the deposition of particles as either a fouling layer or a cake on the surface, the so-called 'particle polarization' effect. This reduces the filtrate or permeate flux through a simple crossflow filter<sup>2-7</sup>. The lateral migration of spherical particles in porous flow channels has also been studied mathematically<sup>8-12</sup>, but rarely have solutions from mathematical models or simulations been compared with experimental measurements to provide any justification of the models proposed. Recent work<sup>12</sup> has indicated that neutrally buoyant rigid spherical particles moving axially along a porous duct in laminar flow experience two radial forces, an inertially induced force and a permeation drag force, which can be added vectorially when

$$\lambda \approx \frac{1}{8} \text{Re}_p \left( \frac{x_{av}}{r_o - r_i} \right)^2 \quad (1)$$

where  $\lambda$  is the ratio of the wall permeation velocity to the maximum axial velocity of the feed suspension to the annulus,  $\text{Re}_p$  the particle Reynolds number,  $x_{av}$  the particle diameter, and  $(r_o - r_i)$  the width of the annular gap. Whilst ultrafiltrations frequently obey criterion (1) microfiltrations are also often close to it, particularly when the particles are finer than 10  $\mu\text{m}$  and the slightest extent of fouling has occurred. Typical microfiltrations, although the feed flow over the filtering surface is

often stated to be turbulent, are in either the laminar or inertial flow regimes. For this reason criterion (1) is perhaps only loosely correct in the context of microfiltration. However, the model developed<sup>12</sup> does appear to be a reasonable starting point to model the process.

## ELECTROKINETIC PHENOMENA TO AID FILTRATION PROCESSES

Electrokinetic phenomena have not been widely used as an aid to liquid filtration, although their possibilities are gradually being researched and recognised. Modified batch filtration equations have been used to model electrofiltrations in a 'deadend' filter<sup>13-15</sup>, and the combined effects of crossflow and electrofiltration and ultrafiltration have been studied<sup>16-19</sup>. Lee<sup>17</sup> is probably the first author to analyse the clear boundary layer which forms at the filtering surface, and also to use a tubular electrode arrangement similar to that used in the present work. The 'deadend' filter analyses<sup>13-15</sup> consider a critical potential to exist at which no particle deposition occurs at the filtering surface; in this arrangement the absence of crossflow precludes the possibility of a clear boundary layer formation. In a crossflow device there is probably a contribution by both the boundary layer and the critical voltage to the prevention of particle deposition.

This paper presents a method for calculating the critical voltage for any given set of operating conditions by calculating the trajectory of a particle entering the annular space between the electrodes. The model has been developed to allow for interpretation of experimental results obtained from a crossflow electrofilter.

## BASIS OF THE MODEL

The motion of a particle suspended in a liquid is largely influenced by two parameters, the motion of the liquid through the specific geometric form of the container and the strength of the imposed electric field.

Referring to Figure 1, the annulus between the two cylinders constitutes the volume through which the slurry flows. When the cylinder walls are non-porous an expression is known for the fluid axial velocity profile (there is no radial fluid motion). When the inner cylinder is porous there is an imposed radial motion which affects the axial motion of the fluid. To calculate the axial and radial velocity profiles it is initially assumed that the permeate or filtrate flux is uniform along the length of the filter, and that the filtrate rate is equal to the convective flow into the tube wall. Assuming that fully developed axial flow exists along the separator (in the design of equipment used in this work entrance effects were minimised and the annular gap width was very small relative to the separator length), and that no radial pressure gradient exists (convective flow into the tube wall is three to four orders of magnitude smaller than the crossflow rate), the equations of motion are then written as:

$$\frac{\partial^2 v_r}{\partial r^2} + \frac{\partial v_r}{\partial r} \left( \frac{1}{r} - \frac{v_r}{v} \right) - \frac{v_r}{r^2} = 0 \quad (2)$$

$$\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} = \frac{v_r}{v} \frac{\partial v_z}{\partial r} + \frac{1}{v} \left( gz - \frac{1}{\rho} \frac{\partial p}{\partial z} \right) \quad (3)$$

These are solved subject to the boundary conditions:

$$\begin{array}{lll} r = r_i & v_r = v_w & v_z = 0 \\ r = r_o & v_r = 0 & v_z = 0 \end{array} \quad (4)$$

to give the radial and axial fluid velocity distributions. Eq. (4) assumes zero slip of fluid at the septum surface.

When the volume between the conducting electrodes is filled by a suspension, the distribution of the electric potential,  $\varphi$ , can be evaluated from Laplace's equation

$$\nabla^2\varphi = 0 \quad (5)$$

when it is assumed that the slurry is dilute and the electric potential is not distorted by the flow of the suspension. The exact solution to eq. (5) can be differentiated to give the field strength between the two cylinders as a function of radius:

$$E = \frac{d\varphi}{dr} = \frac{a}{r} \quad (6)$$

### Particle Motion Through the Separator

Once the fluid flows within the annulus and the distribution of the electric field are known, the trajectory of a spherical particle introduced into one end of the annular space can be calculated. For dilute suspensions the motion of a single particle can be considered, and the small particles encountered in ultrafiltrations and microfiltrations can be considered to be convected by the liquid flowing in the axial direction. By applying Newton's second law to one particle an equation describing its motion under Stokesian conditions in the radial direction can be written:

$$m \frac{du}{dt} = F_e - F_d = qE - 3\pi\mu x_{av} u \quad (7)$$

where  $u$  is the relative velocity between the particle and the fluid,  $F_e$  the electrical force acting on the particle and  $F_d$  the drag force. A negligible error is incurred if the particle motion is considered steady and under zero electric field conditions ( $E = 0$ ) there is no slip at the particle surface whence the equation of motion reduces to:

$$qE = 3\pi\mu x_{av} u = 3\pi\mu x_{av} (v_p + v_r) \quad (8)$$

where  $v_p$  is the particle velocity. Rearranging eq. (8) and substituting for  $E$  from eq. (6) gives:

$$v_p = \frac{dr}{dt} = \frac{q(\varphi_o - \varphi_i)}{3\pi\mu x_{av} \ln(r_o/r_i)} \frac{1}{r} - v_r \quad (9)$$

where the charge on the particle is given by:

$$q = 2\pi\epsilon_0 D x_{av} \zeta (1 + 0.5\kappa x_{av}) \quad (10)$$

where  $\epsilon_0$  is the permittivity of a vacuum,  $D$  the dielectric constant of the fluid,  $\zeta$  the zeta potential, and  $\kappa^{-1}$  the double layer thickness. It is assumed that  $0.5\kappa x \ll 1$  in the present work.

Eq. (9) can be used in combination with eqs. (2) and (3) to calculate the radial position of the particle at any time. At radii closer to the septum there is a greater likelihood of impingement of the particle on the membrane surface and thereby of fouling; once particles have touched the surface it is considered that only some of them will be removed from the flow.

It is important to note in the above that electrophoretic relaxation effects have not been included. Altena and Belfort<sup>12</sup> included in their analysis an inertial or lift velocity as a function of position in the flow domain; this has been omitted from the present work as its magnitude is small compared with the electrical and drag induced velocities for particles of the sizes typically found in micro- and ultra- filtrations.

### Computational Procedure

For computational purposes the annular flow region is divided into a number of radial (usually 1500) and axial distance increments. A general flowchart of the calculational procedures is shown in Figure 2. After having solved the Navier-Stokes equations for the fluid velocity profiles, the feed entry radius in the annulus from which particles are not drawn on to the septum is calculated. All particles entering the separator inside this radius are potentially separable, and their axial point of impingement on the septum surface (and hence probable point of separation) is calculated. The number of particles entering the separator in a radial distance increment is obtained from the feed concentration and the solids flow rate at that radius, and knowing the axial point of impingement of this number, an estimate of the fractional reduction of permeate flux along the separator length is facilitated. The average permeate flux along the septum,  $v_w$ , is then obtained and compared with the experimental flux,  $v_{we}$ :

$$f = 1 - \frac{v_{we}}{v_w} \quad (11)$$

The ratio is related to the number of blinded pores in the septum; in a rudimentary sense it may be considered that the permeate flux is proportional to the number of open pores in the membrane,  $n$ , and hence that  $(v_{we}/v_w) \propto (n_e/n)$ . If  $v_w > v_{we}$ , that is, at any time if the calculated flux is greater than the experimental flux, or the number of open pores by calculation is greater than the open number according to experiment, then it is implied that more particles than are calculated are actually entrained onto the septum surface. Conversely, when  $v_w < v_{we}$  not all particles that touch the septum surface adhere to it.

### DISCUSSION

Results from the mathematical model can be obtained to show effects of operating parameters on axial and radial fluid velocity profiles and on the trajectory of particles passing through the filter unit. Figures 3 and 4 show the calculated radial and axial developed velocity profiles for the size of filter and housing used in this work; these results are a solution of eqs. (2)-(4), noting that a no-slip condition is assumed in the axial direction at the filter surface. The permeate and the crossflow velocities are typical of those found in practice; as might be expected greater permeate rates lead to movement of the maximum crossflow velocity towards the filter septum as a result of a more pronounced effect on the radial velocity profile over the entire annular section. The increased permeate rate pulls a larger proportion of the suspended matter on to the septum surface, thereby increasing the likelihood of potential foulants touching the septum. The fluid velocity profiles have a marked effect on the trajectories of particles fed at different radii across the annular section. Figure 5 shows the trajectories of particles injected at two different radii, with and without imposed electric fields. Particles fed at a location close to the porous septum are pulled onto the septum surface after having travelled about half the length of the separator (the distance travelled is specific to the particular conditions quoted - also, the particle size quoted is a mean size and the effect of particle size distribution is to spread particles over a greater length of the surface). As the field gradient is increased the particles are kept in suspension for a greater part of their travel by the induced electrophoretic velocity. However, under the stated conditions particles fed at a position half way across the annulus show no tendency to approach the separator surface. There

appears to be some indication that it may only be particles which are fed at a point close to the surface which create a fouling problem.

On Figure 6 are experimental results which show the typical effect of increasing the electric field on the filtrate flow rate. Under the conditions of the experiments quoted the filtrate rate falls to about 20% of its initial value within 30 minutes of operation. Imposing an electric field of  $98 \text{ V cm}^{-1}$  causes fewer particles to deposit and the rate falls to just under 50% of its initial value. These are the type of results which would have been expected from earlier studies<sup>14,15</sup>, and demonstrate that an electrical field can facilitate higher filtrate fluxes in crossflow filters. Similar results are available for other solids and over a wide range of experimental conditions<sup>20</sup>.

The typical experimental effects of slurry concentration are indicated in Figure 7. Filtering more dilute slurries causes an equilibrium filtrate rate to be established faster, after which little or no further fouling of the surface occurs.

The pore blocking factor,  $f$ , is positive when more particles than are calculated by the model are deposited at the septum surface, and is negative when some of the particles fail to adhere to the surface. Although a mean particle size has been used in this work it is recognised that when a large proportion of sub-micron particles exist in an aqueous suspension the use of eq. (10) with  $0.5\kappa x \ll 1$  may be questionable, and it is noted that assuming  $0.5\kappa x \gg 1$  would give rise to some effects of particle size distribution. The equation is more applicable to electrophoretic motion in non-aqueous media of low conductance. However, evaluation of  $\kappa$  *a priori* can be problematical and to use a predetermined value of  $\kappa$  is unlikely to offer any more precise a result in such a process as described here, although it may be more fundamentally acceptable. Furthermore, the applied electric field and the field of the electric double layer are implicitly assumed to be simply superimposed, but mutual distortion of these fields may affect electrophoretic mobility<sup>21</sup> through abnormal (surface) conductance in the vicinity of the charged surface and through loss of double layer symmetry (relaxation effects). To some extent, therefore, the blocking factor may also be serving to correct the model for the above noted factors which are themselves somewhat unquantifiable for anything but an idealized particle.

The pore blocking factor passes through a maximum value,  $f_{max}$ , when plotted against the filtration time. If  $f_{max}$  is then used to normalize the factor as  $f/f_{max}$ , and then plotted against filtration time a general curve of the type plotted on Figure 8 results when the feed concentration is low ( $< 0.009\%$  v/v for anatase). The maximum factor value is dependent primarily on feed slurry concentration, the applied electric field, and the crossflow velocity.

## CONCLUSIONS

Fine particles separated in ultra- and micro- filtrations can contribute to, and sometimes be the sole cause of, membrane fouling. The particles entering a crossflow electro-separator are subject to electrical and drag induced velocities; others such as the inertial or lift velocity are orders of magnitude smaller for such fine particles. Imposing a d.c. electric field across the membrane can serve to facilitate the maintenance of a high filtrate flux, preventing much of the fouling which results in the absence of the field. A model has been described which gives good qualitative descriptions of crossflow separations when calculations are compared with experiments, and this is adequate to form the basis of a model for the engineering appraisal of the use of electrical fields to assist crossflow filtration.

## ACKNOWLEDGEMENT

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## NOTATION

$a$	constant resulting from integration of eq. (5) (V)
$D$	dielectric constant of the fluid
$E$	electric field ( $V\ m^{-1}$ )
$f$	pore blocking factor defined by eq. (11)
$g$	acceleration due to gravity ( $m\ s^{-2}$ )
$l$	length of separator (m)
$m$	mass of a particle (kg)
$n$	number of open pores in membrane (by calculation)
$n_e$	number of open pores in membrane by experiment
$q$	charge on a particle (C)
$r$	radial coordinate (m)
$r_i$	radius of inner electrode (m)
$r_o$	radius of outer electrode (m)
$Re_p$	particle Reynolds number, ( $x_{av}v_z/v$ )
$t$	time (s)
$u$	particle velocity relative to the fluid ( $m\ s^{-1}$ )
$v_p$	particle velocity ( $m\ s^{-1}$ )
$v_r$	radial velocity ( $m\ s^{-1}$ )
$v_w$	permeate flux ( $m^3\ m^{-2}\ s^{-1}$ )
$v_{we}$	permeate flux by experiment ( $m^3\ m^{-2}\ s^{-1}$ )
$v_{wi}$	permeate flux at start of filtration ( $m^3\ m^{-2}\ s^{-1}$ )
$v_z$	axial velocity ( $m\ s^{-1}$ )
$x$	mean particle diameter (m)
$z$	axial coordinate (m)

## Greek symbols

$\epsilon_0$	permittivity of a vacuum ( $C^2\ J^{-1}\ m^{-1}$ )
$\kappa$	reciprocal double layer thickness ( $m^{-1}$ )
$\lambda$	ratio of wall permeation velocity to maximum axial velocity of feed suspension to annulus
$\mu$	fluid dynamic viscosity (Pa s)
$\nu$	fluid kinematic viscosity ( $m^2\ s^{-1}$ )
$\rho$	fluid density ( $kg\ m^{-3}$ )
$\phi$	electric potential (V)
$\phi_i$	electric potential at inner electrode (V)
$\phi_o$	electric potential at outer electrode (V)
$\zeta$	zeta potential (V)

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## TABLES AND FIGURES

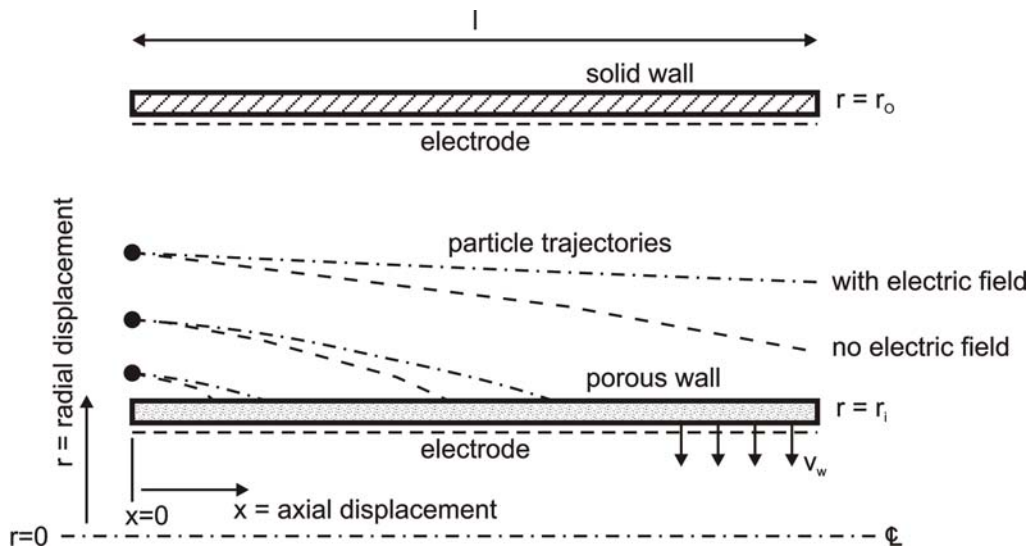


Figure 1: Particle and liquid flows through an annulus with a porous inner wall.

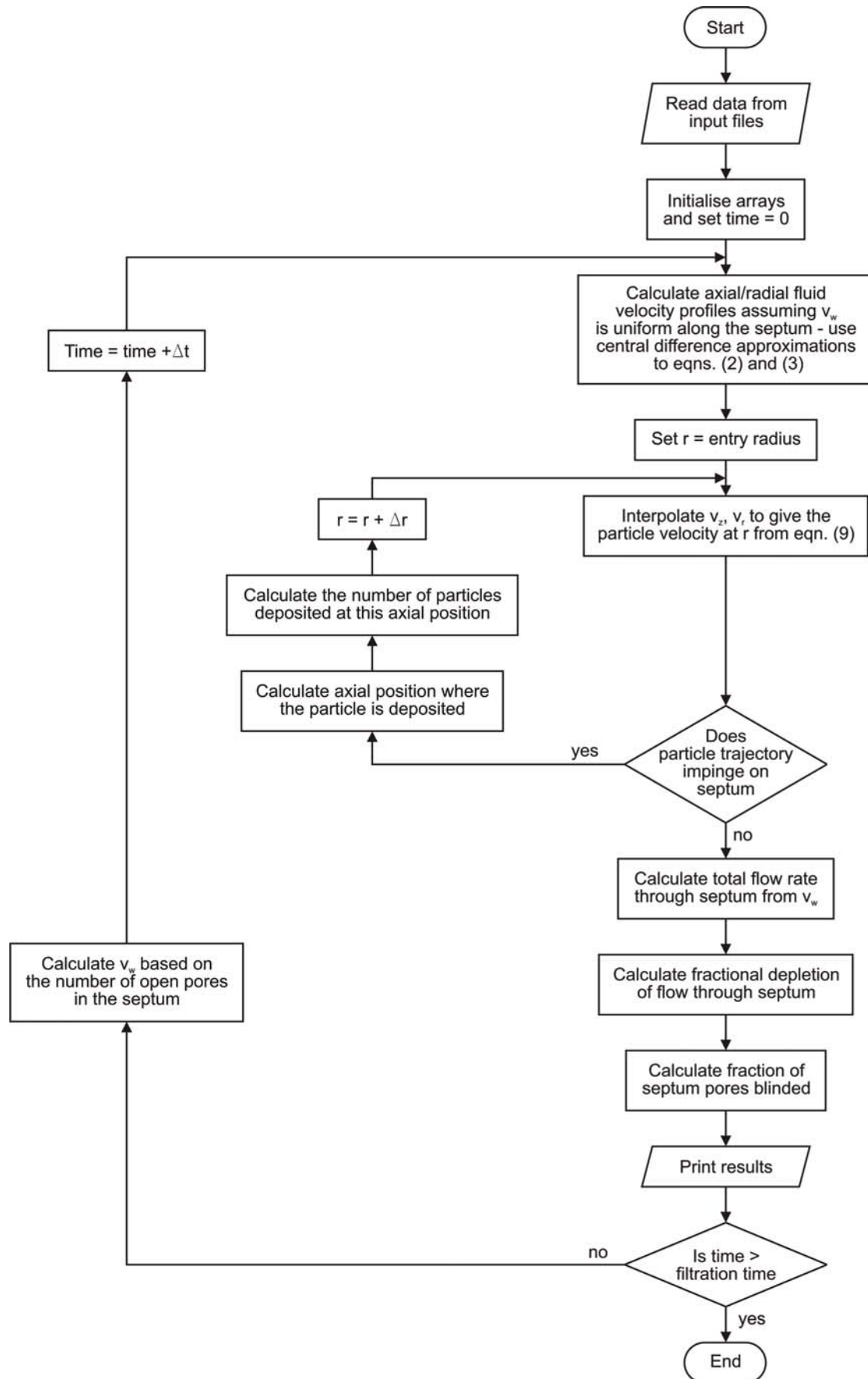


Figure 2: General flowchart of the calculational procedure to model membrane fouling rates.

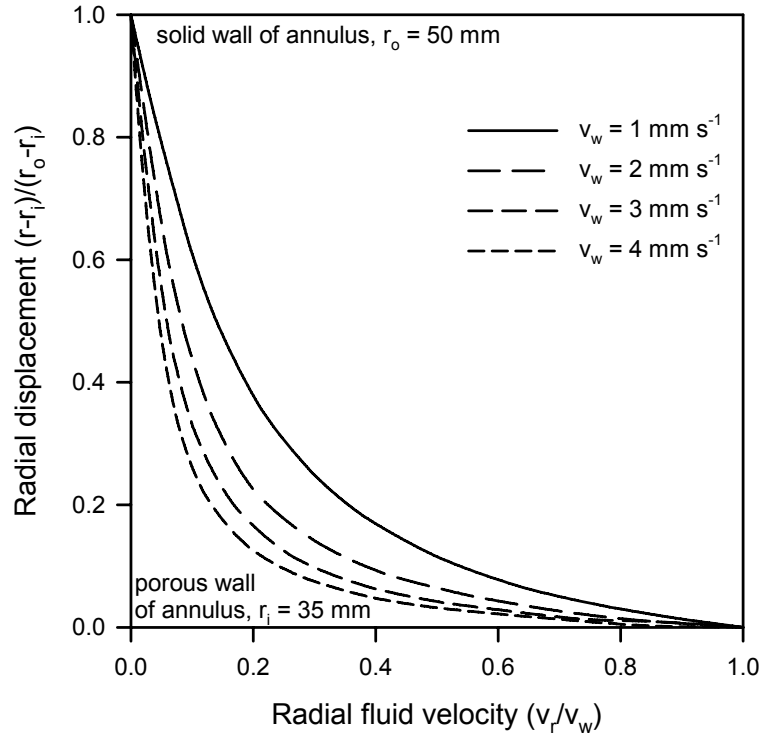


Figure 3: Effect of suction at inner wall on radial velocity profiles.

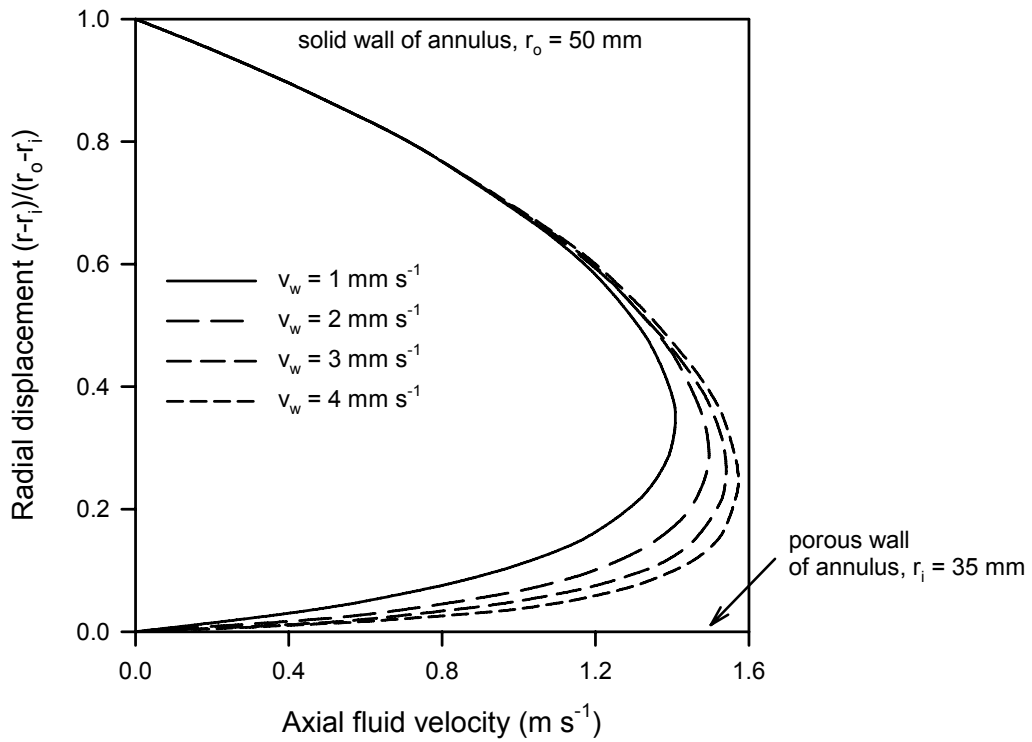


Figure 4: Effect of suction at inner wall on axial velocity profiles.

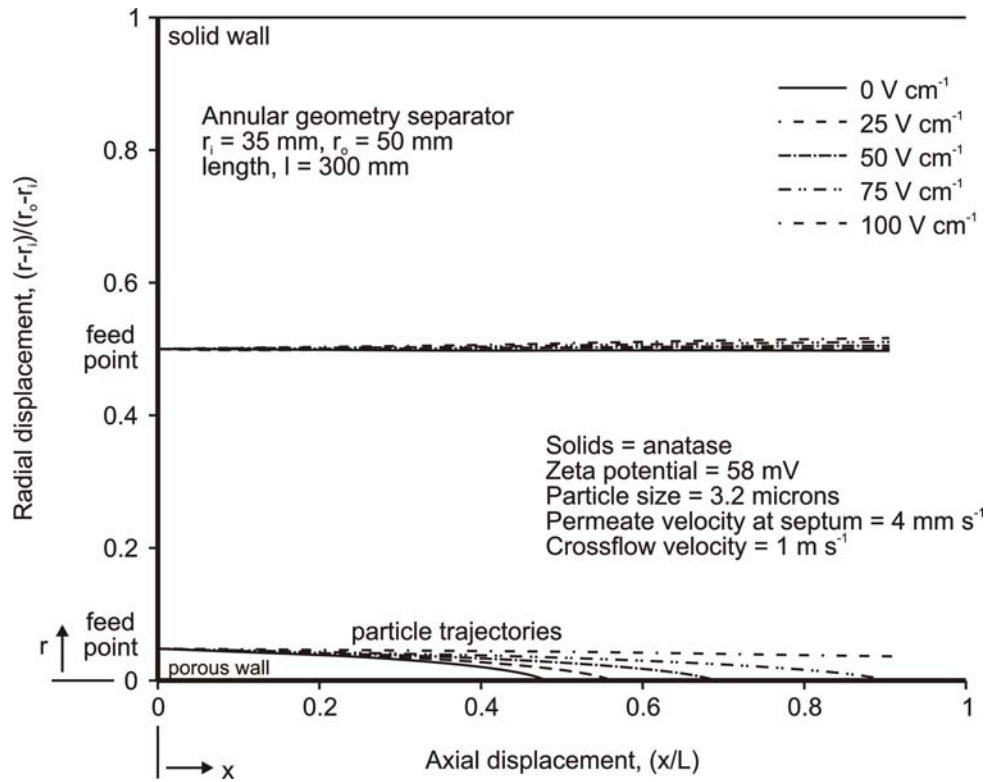


Figure 5: Effect of electric field on particle trajectory.

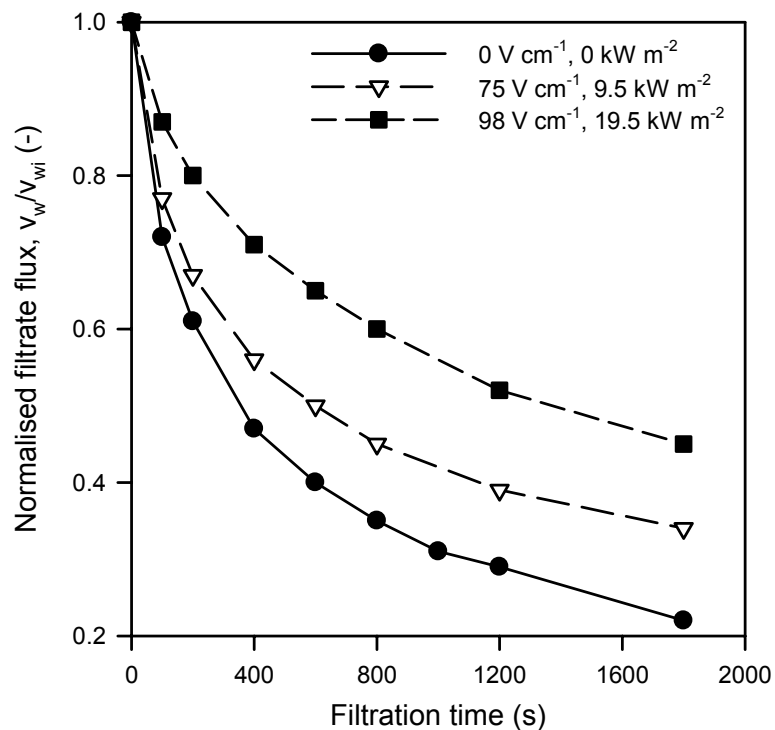


Figure 6: Effect of electric field on permeation rates during crossflow filtration of a china clay suspension. Feed concentration = 0.04% by volume; Crossflow velocity = 0.9 m s<sup>-1</sup>; Initial filtration rate = 2.18 m<sup>3</sup> m<sup>-2</sup> h<sup>-1</sup>.

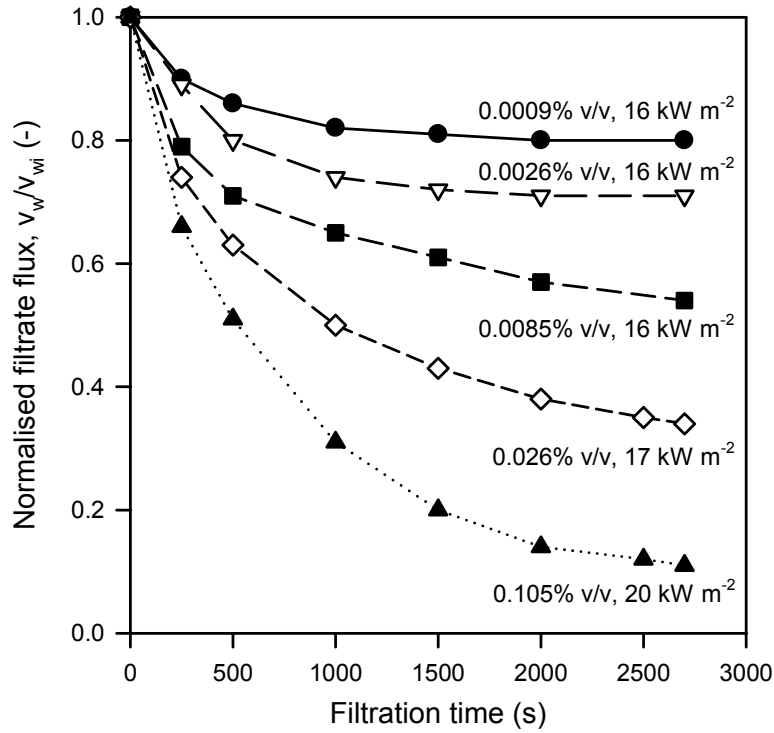


Figure 7: Effect of slurry concentration on permeation rates during crossflow filtration of anatase suspensions. Crossflow velocity =  $0.9 \text{ m s}^{-1}$ .

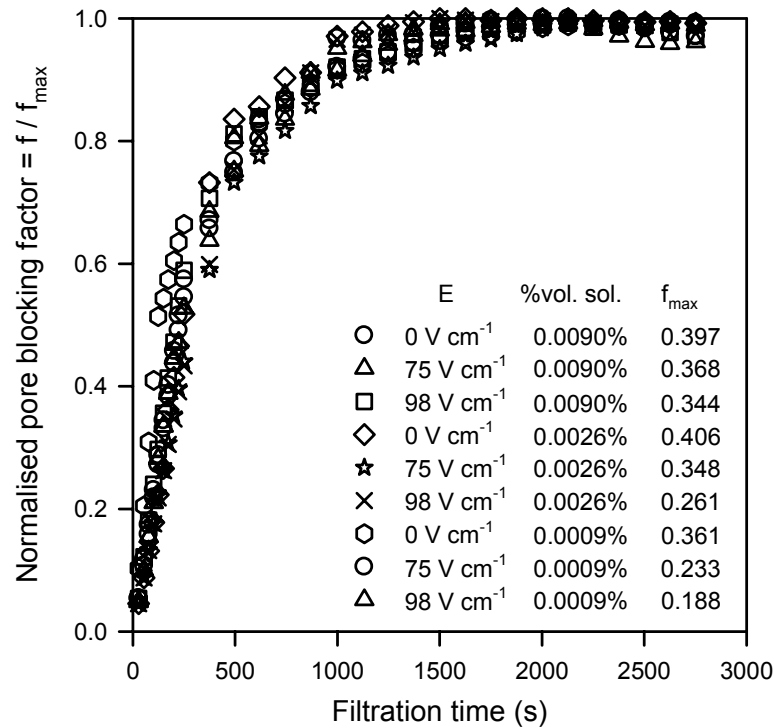


Figure 8: Variation of pore blocking factor during the crossflow filtration of anatase filtration. Crossflow velocity =  $0.9 \text{ m s}^{-1}$ .