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    - Provide a Conclusions section toward the end of the paper.
    - Any mathematical expressions should be typed and checked carefully for accuracy. Where several equations appear, a list of symbols used should be inserted at the end of the paper (before any References). SI units should always be used.
    - References should be listed in the order in which they are first cited in the text, where they are indicated by the superscript numbers. They should give in order: Author's surnames and initials, Year, Title of paper/chapter, Title of book or journal, Volume, Issue, Page number, followed by the name and town of the publisher in the case of a book.

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FILTRATION OF FIBRE/PARTICLE MIXTURES

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New data are reported for the filtration of binary fibre/titania (rutile) mixtures. The combined use of the Kozeny-Carman equation and Darcy's law is discussed in relation to the filtration of these mixtures. Upon study this approach is shown to have its limitations, particularly when significant aggregation takes place between the two solids species. The difficulties in overcoming these limitations from a fundamental basis are highlighted and a semi-empirical model is presented and discussed. This model is shown to better represent the specific resistance trend with solids composition for a range of binary mixture filtration data.

INTRODUCTION

Previous works on the cake filtration of binary mixtures include those of Shirato *et al.*[1], Abe *et al.*[2,3], Wakeman[4], Iritani *et al.*[5] and Chellappah *et al.*[6-8]. Although considerable work has reported on the effects of physico-chemical interactions on filtration behaviour[9-12], as Iritani *et al.*[5] point out, there is scarce study on the effects of physico-chemical factors in binary suspension filtration. There is also sparsity in the study of the filtration of fibre/particle mixtures, although the need to dewater such mixtures often arises in industry.

In relation to design and scale-up, the permeability of a filter cake is the most important factor in cake filtration and is often interpreted through a measure of specific resistance. Most models for cake formation are based on Darcy's law, which uses a single parameter (the permeability) to account for characteristics of the (porous) cake in so far as they affect fluid flow. There have been many attempts to relate permeability to geometric considerations of a porous medium, including the widely used theory of Kozeny-Carman. Here, fluid flowing through a porous body is assumed to lose energy where it is in contact with the internal surfaces; these internal surfaces often possess a very complex geometry which cannot be easily described mathematically, but for practical purposes can be interpreted through porosity and specific surface. The Kozeny-Carman equation has important uses for determining the specific surface of particle samples from permeability data and in correlating resistance data for fluid flow through porous media[13].

In this paper physico-chemical interactions in binary fibre/titania (rutile) mixtures are highlighted. The combined use of the Kozeny-Carman equation and Darcy's law is presented and discussed in relation to their application in the filtration of binary mixtures. A semi-empirical equation is also proposed to represent aggregation phenomena and/or changes in packing behaviour with binary filter cakes.

THEORETICAL DEVELOPMENT

Abe *et al.*[2] have previously derived a model to predict the average specific cake resistance of binary mixtures based on Darcy's law and the Kozeny-Carman equation. The model derivation is briefly presented here with further development and relevant elucidation. The Kozeny-Carman equation can be written as:

$$\Delta P = \frac{5S_m^2\left(1-\varepsilon_{av}\right)^2}{\varepsilon_{av}^3}Lu\mu \tag{1}$$

and Darcy's law, where the pressure drop across the filter cake (ΔP) can also be expressed as:

$$\Delta P = u\mu\alpha_{av}w \tag{2}$$

where S_m is the specific surface of solids, ε_{av} the average cake porosity, u the superficial filtrate velocity, μ the liquid viscosity, α_{av} the average specific cake resistance and w the mass of solids per unit area. Also, the height of filter cake (L) is given by:

$$L = \frac{v}{1-\varepsilon_{av}} \tag{3}$$

and v/w is the inverse of the binary filter cake true density (ρ_m), expressed as:

$$\frac{v}{w} = \frac{1}{X_D\rho_{s1} + \left(1-X_D\right)\rho_{s2}} \tag{4}$$

where v is the true volume of cake per unit area, X_D the volume ratio of larger solids to total solids in the mixture, ρ_s the solids density and the subscripts 1 and 2 refer to the larger solids (fibres) and smaller solids (rutile), respectively. Substituting equations (2)-(4) into

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(1) and manipulating gives:

$$\alpha_{av} = \frac{5\left(1-\varepsilon_{av}\right)}{\varepsilon_{av}^{3}\left(X_D\rho_{s1}+\left(1-X_D\right)\rho_{s2}\right)}S_m^2 \tag{5}$$

Assuming an 'additive' mechanism, the specific surface of the binary mixture at a given solids composition can be expressed in terms of the effective specific surface of the two solids components:

$$S_m = X_D\frac{S_1}{\varphi_1}+\left(1-X_D\right)\frac{S_2}{\varphi_2} \tag{6}$$

Equation (6) assumes that with mixtures the pure component specific surfaces remain fixed and any changes in total specific surface are due only to changes in volume of the two solids species. The equation does not account for any changes in packing and/or interaction effects of the solids present. The terms S_1/φ_1 and S_2/φ_2 describe the specific surface of the pure fibres and rutile in contact with the fluid permeating through the cake as determined from the pure component filtration data:

$$\frac{S_i}{\varphi_i} = \sqrt{\frac{\alpha_{av,i}\varepsilon_{av,i}^{3}\rho_{si}}{5\left(1-\varepsilon_{av,i}\right)}} \tag{7}$$

No relationship between specific surface and particle size is assumed and φ_i is termed a shape factor. Also, S_i/φ_i is kept as a ratio and not broken up since little is known about the exact relative magnitudes of either S_i or φ_i. φ_i can be thought of as a lumped parameter that takes into account sphericity, surface roughness, as well as pure component size distribution and surface interactions. Hence, S_i can be considered to be the specific surface that corresponds to idealised spheres. As pointed out by Donohue and Wensrich[14], although a shape factor can be used to account for irregular particles, it must be determined empirically. Furthermore, the ratio of the two solids specific surfaces (S_B) is a recurring term and given by:

$$S_B = \frac{S_1/\varphi_1}{S_2/\varphi_2} \tag{8}$$

It should be noted that S_B in equation (8) refers to the ratio of pure component specific surfaces, with the specific surface of a pure component determined from equation (7). However, the effective specific surface of a pure component may vary in the presence of another solids component due to physico-chemical effects and/or varying packing mechanisms.

MATERIALS, APPARATUS AND EXPERIMENTAL METHOD

The fibres used in the experiments were in the form of tissue paper (Merton Cleaning Supplies, Leicester). The titania was in the form of the rutile polymorph and obtained from Huntsman. Some relevant properties of the fibres and rutile are shown in Table 1 where the ζ-potential values quoted are over the pH range of interest. It is evident that the fibres display a much wider size distribution than rutile. The ζ-potential data suggest Na^+ to be an indifferent electrolyte and indicate physical adsorption of Ca^{2+} ions onto the fibre and rutile surfaces. The ionic strength (I) of 0.2 M NaCl and 0.1 M $CaCl_2$ solutions were 0.2 and 0.6, respectively, as calculated using the Debye-Hückel theory[15]:

$$I = \frac{1}{2}\sum_j z_j^{\,2} b_j \tag{9}$$

where z_j is the charge number of ion j and b_j its concentration. Well mixed suspensions consisting of just fibres, just rutile or binary mixtures of the two in deionised (DI) water, 0.2 M NaCl solution or 0.1 M $CaCl_2$ solution were used. In both filtration and sedimentation experiments the total solids concentration was maintained at 1.1% v/v. Typical SEM images of the rutile and fibres, as well as more detailed descriptions of solids characterisation and suspension preparation have been previously reported[6,7].

An automated pressure filtration apparatus was used for the filtration experiments, details of which have been described elsewhere[16,17]. Further details of the filtration and sedimentation experiments, including methodology and some initial results, have been previously reported[6].

EXPERIMENTAL RESULTS AND DISCUSSION

Filtration experiments were performed at constant

	10, 50 & 90 % sizes[a] (μm)	Aspect ratio	ζ-potential (mV)		
			DI water	0.2 M NaCl	0.1 M $CaCl_2$
Rutile	0.4, 0.45 & 0.7	2-4	-35	-30	+21
Fibres	6, 15 & 29	~100	-18	-20	-5

[a] Width for fibres

Table 1: Relevant properties of the two solids used.

pressure and analysed to give values of average specific cake resistance (α_{av}) and cake porosity (ε_{av}). Sedimentation experiments were analysed to give values of initial settling rate and proportion of sludge. The effects of solids composition on binary suspension filtration are presented in terms of X_D which is defined as the ratio of the volume of fibres to the total volume of solids in the suspension; $X_D = 0$ indicates a pure rutile suspension (i.e. no fibres) whereas $X_D = 1$ indicates a pure fibre suspension (i.e. no rutile).

Figure 1 shows some typical sedimentation data used to help interpret the filtration results. It is evident that pure rutile and pure fibres in DI water constitute stable suspensions which do not readily settle. With only small amounts of fibres added to a pure rutile suspension, rutile-fibre interactions take place and the suspension de-stabilises. The general sedimentation trends for suspensions in DI water and $CaCl_2$ solution appear to be similar. Initial settling rates in the range $0.1 < X_D < 0.8$ were lower for suspensions in $CaCl_2$ solution, presumably due to the formation of low density, loosely networked aggregates as evidenced by the greater proportions of sludge at a given X_D. Also, it is not surprising that the proportion of sludge data suggests that the fibres form more loosely networked packings than rutile.

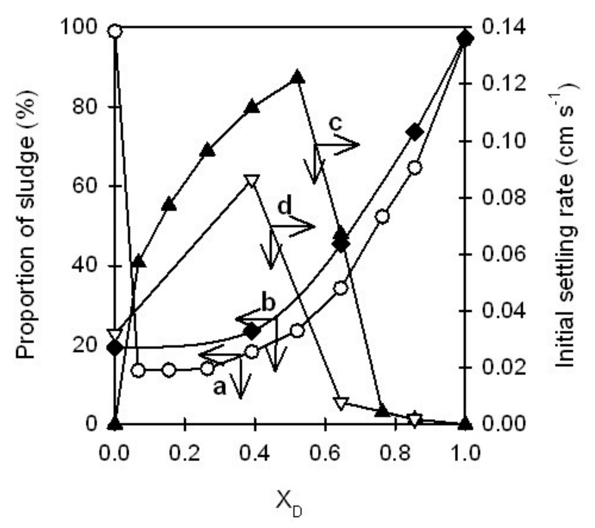

Figure 1: Effects of fibre fraction (X_D) on the proportion of sludge and initial settling rate (a = proportion of sludge in DI water; b = proportion of sludge in 0.1 M $CaCl_2$; c = initial settling rate in DI water; d = initial settling rate in 0.1 M $CaCl_2$).

Figures 2 and 3 show the effects of solids composition and pressure on ε_{av} and α_{av} for DI water filtrations. Figures 4 and 5 show corresponding data for solids composition and solution environment at 450 kPa with DI water, 0.2 M NaCl and 0.1 M $CaCl_2$. It is evident that the effects of pressure are less significant compared to those observed with variations in solids composition and solution environment. Some discussion on the effects of solids composition and filtration pressure have been previously published by the authors[6], where the minima in α_{av} over the solids composition range $0.3 < X_D < 0.7$ (Figure 3) was attributed to rutile-fibre aggregation. Rutile-fibre aggregation was noted visually and supported by sedimentation data. The α_{av} vs. X_D trend for suspensions in DI water generally corresponds to the trend of initial settling rate vs. X_D (Figure 1), and this is not surprising as the initial settling rates were largely influenced by the degree of aggregation.

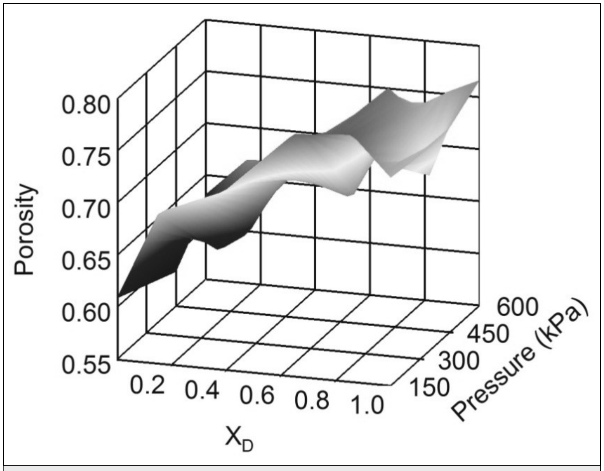

Figure 2: Effects of fibre fraction (X_D) and pressure on ε_{av} for filtrations from DI water.

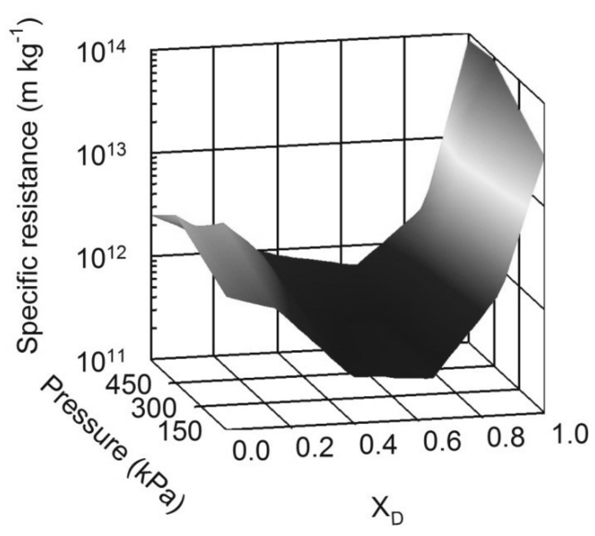

Figure 3: Effects of fibre fraction (X_D) and pressure on α_{av} for filtrations from DI water.

The relationships between α_{av} and ε_{av} with fibre fraction give rise to an apparent anomaly, since, as sug-

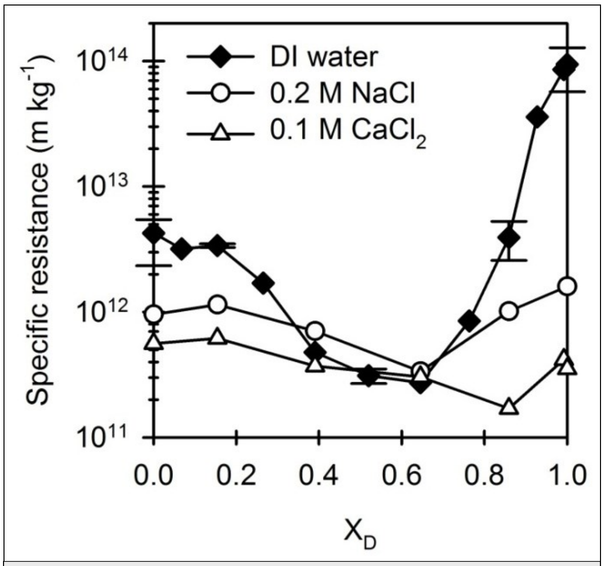

Figure 4: Effects of solids composition and solution environment on α_{av} for 450 kPa filtrations.

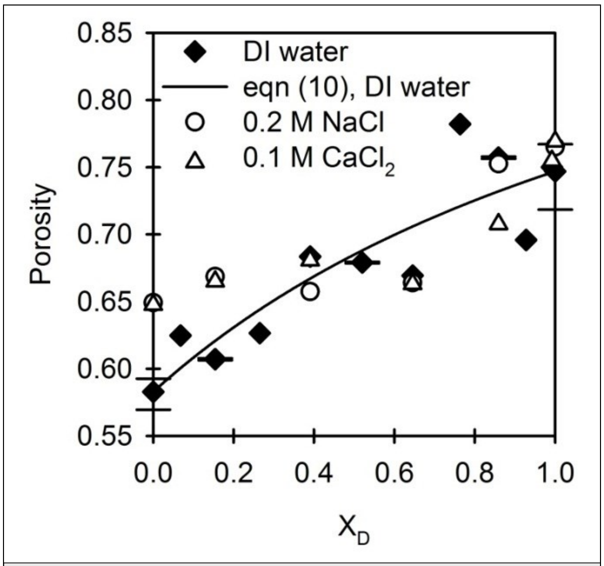

Figure 5: Effects of solids composition and solution environment on ε_{av} for 450 kPa filtrations. The prediction given by the additive porosity model (equation (10)) is included for filtrations from DI water.

gested by the Kozeny-Carman and Happel cell models[13], an increase in porosity generally results in an increased permeability (i.e. lower α_{av}). However, it should be recognised that these models only provide reasonable approximations if the solids forms remain similar. The fibres and rutile are of very different forms, with the rutile being more isometric in shape, and this may result in packing variations when moving from pure rutile to pure fibre cakes. Also, the rutile-fibre interactions may further influence the specific surface exposed to the fluid flow at the various solids compositions, which in turn will affect specific cake resistance.

In terms of packing, the additive porosity model[1] gives reasonable representations of porosity data for filtrations from DI water:

$$\varepsilon_{av} = \frac{\dfrac{\varepsilon_{av,1} X_D}{1-\varepsilon_{av,1}} + \dfrac{\varepsilon_{av,2}\left(1-X_D\right)}{1-\varepsilon_{av,2}}}{\dfrac{X_D}{1-\varepsilon_{av,1}} + \dfrac{1-X_D}{1-\varepsilon_{av,2}}} \qquad (10)$$

Figure 5 shows the trend of ε_{av} vs. X_D given by equation (10) for 450 kPa filtrations from DI water. The additive law represented the experimental porosity data better than typical inter-particle penetration models which generally showed a minimum porosity at an intermediate mixture fraction. This perhaps indicates that rutile particles coat the fibre surfaces, thereby reducing the inter-fibre porosity to a small extent, instead of more intensively filling up inter-fibre voids[6].

In general, the 0.1 M CaCl$_2$ solution environment had a marginally greater effect on filtration behaviour compared to 0.2 M NaCl. This could be due to:

1. Ca^{2+} ions physically adsorbing onto the solids surfaces whereas Na$^+$ ions behaved as an indifferent electrolyte
2. The 0.1 M CaCl$_2$ solution being of a greater ionic strength.

Taking into account that the fibres and rutile were oppositely charged in 0.1 M CaCl$_2$ solution, the effect of the CaCl$_2$ solution environment on rutile-fibre interactions could be expected to be greater than on the pure component interactions. However, there was little variation in ε_{av} (Figure 5) and α_{av} (Figure 4) due to the different solution environment in the solids composition range $0.3 < X_D < 0.7$. The apparent discrepancy may be explained by the fact that increasing states of aggregation cannot be expected to continually decrease α_{av} and increase ε_{av} as there will eventually be a limit for a given applied pressure. The limit seems to have already been reached in cakes formed from DI water suspensions over the range $0.3 < X_D < 0.7$. Hence, it is postulated that up to a certain extent interaction (aggregation) between the two solids components will strongly affect a binary cake's average specific resistance due to a decrease in specific surface and/or increase in porosity, with the magnitude of the effects varying with solids composition and solution environment.

MODELLING ANALYSIS

Before showing application of the model outlined earlier to experimental data, some general characteristics are discussed. In particular, the potential for representing a minimum in specific resistance at an intermediate solids composition and the significance of aggregation effects.

Study of Model Characteristics

Considering equations (5) and (6), and assuming the hypothetical case of constant ε_{av} and that S_1/φ_1 and S_2/φ_2 are invariant with solids composition, equating the differential of α_{av} to 0 (which represents where a minimum occurs), leads to an interesting outcome. With relevant manipulation and the use of equation (8), it is seen that a minimum in α_{av} at an intermediate solids composition can only occur if:

$$\frac{2\rho_{s1}}{\rho_{s1}+\rho_{s2}} < S_B < \frac{\rho_{s1}+\rho_{s2}}{2\rho_{s2}} \qquad (11)$$

Alternatively, taking into account variations in ε_{av} using equation (10), but still assuming constant S_1/φ_1 and S_2/φ_2 (i.e. specific surface is invariant with solids composition) gives:

$$\alpha_{av} = \frac{a_1}{a_2}\left(X_D + (1-X_D)\frac{1}{S_B}\right)^2\left(\frac{S_1}{\varphi_1}\right)^2 \qquad (12)$$

where

$$a_1 = 5\left(1-\left(\frac{\frac{\varepsilon_{av,1}X_D}{1-\varepsilon_{av,1}}+\frac{\varepsilon_{av,2}(1-X_D)}{1-\varepsilon_{av,2}}}{\frac{X_D}{1-\varepsilon_{av,1}}+\frac{1-X_D}{1-\varepsilon_{av,2}}}\right)\right)$$

$$a_2 = \left(\frac{\frac{\varepsilon_{av,1}X_D}{1-\varepsilon_{av,1}}+\frac{\varepsilon_{av,2}(1-X_D)}{1-\varepsilon_{av,2}}}{\frac{X_D}{1-\varepsilon_{av,1}}+\frac{1-X_D}{1-\varepsilon_{av,2}}}\right)^3 \left(X_D\rho_{s1}+(1-X_D)\rho_{s2}\right)$$

From equation (12), and still assuming constant S_1/φ_1 and S_2/φ_2, equating the differential of α_{av} to 0 shows that a minimum in α_{av} only occurs if:

$$a_3 < S_B < a_4 \qquad (13)$$

where

$$a_3 = \frac{1}{\left(1.5\frac{\varepsilon_{av,2}}{\varepsilon_{av,1}}-1\right)\left(\frac{1-\varepsilon_{av,1}}{1-\varepsilon_{av,2}}\right)+0.5\frac{\rho_{s2}}{\rho_{s1}}}$$

$$a_4 = \left(1.5\frac{\varepsilon_{av,1}}{\varepsilon_{av,2}}-1\right)\left(\frac{1-\varepsilon_{av,2}}{1-\varepsilon_{av,1}}\right)+0.5\frac{\rho_{s1}}{\rho_{s2}}$$

Equation (13) is another interesting result as it outlines the range of S_B, in terms of the two pure solids densities and pure component ε_{av}, within which a minimum or maximum in α_{av} could occur at an intermediate solids composition. In principle, the equation is valid for any binary mixture where the effective specific surface values of the two solids components do not vary with solids composition and the ε_{av} vs. solids composition trend is satisfactorily represented by the additive porosity model.

Using data from the 450 kPa filtrations of fibre/rutile mixtures in DI water as an example, equation (11) simplifies to show that a minimum in α_{av} will only occur if $0.47 < S_B < 0.66$. Using the same dataset, equation (13) correspondingly simplifies to $0.58 < S_B < 1.73$, indicating that accounting for the porosity variation widens the limits within which a minimum in α_{av} could be obtained. It is noted that data obtained at other filtration pressures showed similar limits. According to equations (7) and (8), the limits of the expressions represented in equations (11) and (13) are not met experimentally if S_B values are calculated from the pure component filtration data. For instance, using data from filtrations with DI water, S_B was *ca.* 3.7, 4.9 and 5.5 for pressures of 150, 450 and 600 kPa, respectively.

Significance of Aggregation

As an illustrative example, Figure 6 shows a family of curves where first S_1/φ_1 and then S_2/φ_2 is altered such that, in each case, S_B ranges from 0.65 to 1.45. Curves (a), (b) and (c) are plotted using the fixed value of S_2/φ_2 determined from equation (7) with experimental data from a 450 kPa filtration of a pure rutile suspension in DI water; values of S_1/φ_1 were varied to give the required range of S_B. Similarly, for curves (d),

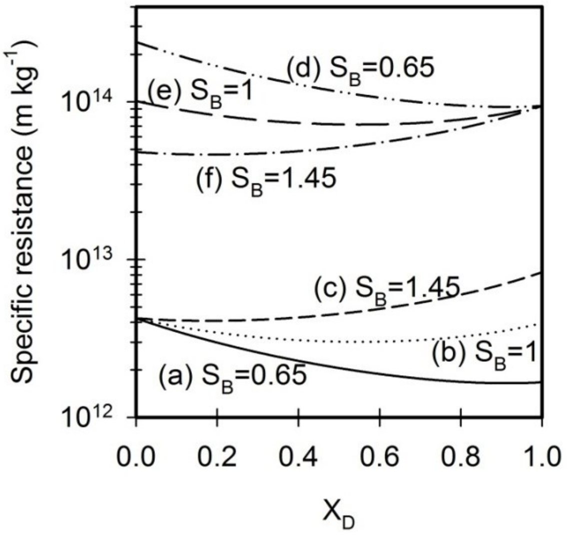

Figure 6: Example family of curves with first S_1/φ_1, and then S_2/φ_2 altered such that S_B falls between 0.65 and 1.45 (the limits set by equation (13) for 450 kPa filtrations from DI water were 0.58 to 1.73).

(e) and (f), S_1/φ_1 was calculated from equation (7) with the same experimental dataset and values of S_2/φ_2 were varied. Values of the pure component porosities were taken from the 450 kPa pure component filtrations with DI water. Since the values of S_B were within the range outlined by equation (13), minima in α_{av} were obtained at an intermediate solids composition. However, these are not as pronounced as the experimentally observed minima (see Figures 3 and 6). Such a discrepancy may not be surprising since, as noticed visually and evidenced by the sedimentation data where both pure component suspensions do not settle but settling rates are significantly increased for mixtures, the physical nature of the binary suspensions are altered synergistically. The consequence is that the behaviour of an interacting binary suspension cannot usually be determined by a simple extrapolation of the known behaviour of the two pure component suspensions.

It should be noted that if the effective specific surface of rutile and/or fibres vary with solids composition due to aggregation and/or changes in packing behaviour, so S_B may also vary with solids composition. In the analysis so far it has been assumed that S_B remains constant and can be determined from the pure component filtrations. The significance of S_B in the analysis of binary mixture filtration is highlighted in equations (11) and (13) and Figure 6. For S_B values to be the basis of a meaningful study, it will have to represent the true ratio of effective specific surfaces at any given solids composition. It is difficult to determine a suitable method of fundamentally accounting for the variation of specific surface of the two solids species with solids composition due to the inherent complexities likely to be introduced by aggregation and varying packing mechanisms. To overcome these limitations, a fitting procedure has been carried out on the total solids effective specific surface.

Model Fitting

Representations of S_m at the different solids compositions were experimentally determined by rearranging equation (5) to give:

$$S_m = \sqrt{\frac{\alpha_{av}\varepsilon_{av}{}^3\left(X_D\rho_{s1}+\left(1-X_D\right)\rho_{s2}\right)}{5\left(1-\varepsilon_{av}\right)}} \qquad (14)$$

Equation (6) gives an ‘additive’ type approach to describing the variation of total effective specific surface with solids composition, and only accounts for the variation in total specific surface brought about by progressively ‘removing’ a given volume of solids component 1 and ‘replacing’ it with solids component 2 of a similar volume. Where the two solids are well mixed and of significantly different size distributions and/or shapes (which is more likely to result in packing variations), and in particular where interactions between the two solids occur, then equation (6) may not be appropriate. Instead, equation (15) can be used to represent the variation of total solids effective specific surface:

$$S_m = a_5 + a_6 \qquad (15)$$

where

$$a_5 = X_D\frac{S_1}{\varphi_1}\exp\left(-b_1\left(1-X_D\right)\right)$$

$$a_6 = \left(1-X_D\right)\frac{S_2}{\varphi_2}\exp\left(-b_2X_D\right)$$

where b_1 and b_2 are fitting parameters. Values of b_1 and b_2 where calculated using non-linear regression on the values of S_m determined from equation (14). S_1/φ_1 and S_2/φ_2 were determined using the pure component filtration data and equation (7).

The exponential functions in equation (15) can be considered to describe variations in total specific surface due to aggregation and variations in packing behaviour. In the absence of a more fundamental model, b_1 and b_2 give quantitative estimates of these effects by behaving as ‘lumped coefficients’. The fitting parameter b_1 accounts for the effects induced by adding small solids on the larger solids component’s effective specific surface. Correspondingly, b_2 accounts for the effects of adding larger solids on the small solids component’s effective specific surface. So, for example, as the cake becomes richer in the small solids component and X_D decreases, then the term ‘$\exp(-b_2X_D)$’ in equation (15) becomes less significant. This indicates a reduced influence of the larger solids component on the small solids component’s effective specific surface. However, the term ‘$(1-X_D)(S_2/\varphi_2)$’ becomes greater due to the increasing volume of the small solids component in the cake. Furthermore, the greater the magnitude of b_2, so the more of an effect the term ‘$\exp(-b_2X_D)$’ will have on the term ‘$(1-X_D)(S_2/\varphi_2).\exp(-b_2X_D)$’.

Substituting equation (15) into equation (5) gives:

$$\alpha_{av} = \frac{5\left(1-\varepsilon_{av}\right)}{\varepsilon_{av}^3\left(X_D\rho_{s1}+\left(1-X_D\right)\rho_{s2}\right)}\left(a_5+a_6\right)^2 \qquad (16)$$

Figure 7 shows the fits given by equation (16) to the filtration data for fibre/rutile mixtures in DI water at 150, 450 and 600 kPa. The model was also fitted to data for 450 kPa filtrations of fibre/rutile mixtures in 0.2 M NaCl and 0.1 M $CaCl_2$ solutions (see Figure 8). In both cases the fits obtained were good.

To further trial the model the 196 kPa filtration data reported by Iritani *et al.* (2002) were used. In their paper suspensions comprising mixtures of rutile and silica in DI water at two different pH values (4.5 and

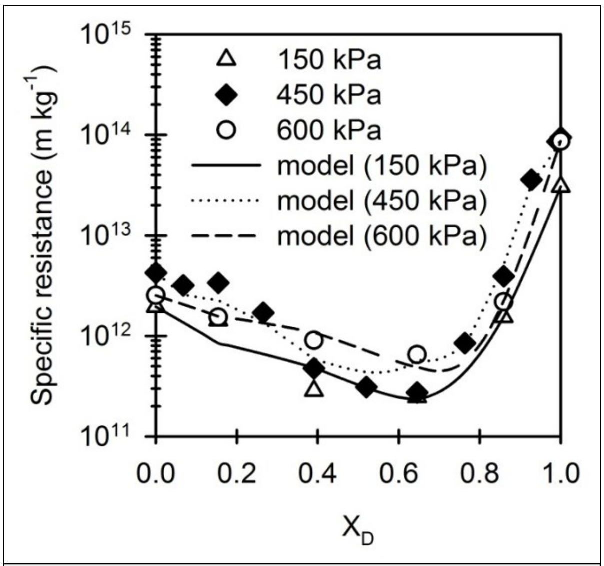

Figure 7: Experimental data for filtrations of fibre/rutile mixtures in DI water at 150, 450 and 600 kPa, and model fits using equation (16).

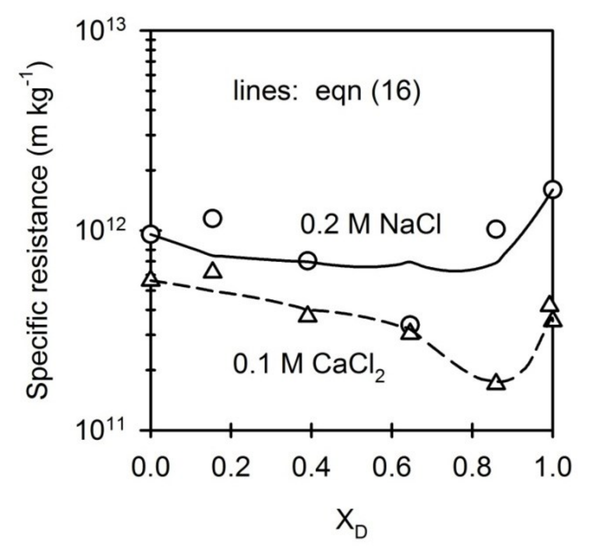

Figure 8: Experimental data for 450 kPa filtrations of fibre/rutile mixtures in 0.2 M NaCl and 0.1 M $CaCl_2$ solutions, and model fits using equation (16).

9.6) were filtered. At pH 4.5 rutile-silica interactions resulted in a minimum in the specific resistance vs. solids composition trend, whereas at pH 9.6 the particles were well dispersed. Hence, the model could also be tested on a different particle/particle binary mixture where aggregation state is known to vary. The resultant fits are shown in Figure 9. To be consistent with the other data reported in the current paper, X_D is defined as the volume ratio of the larger solids component (silica in the case of Iritani *et al*.'s data) to the total solids in the suspension. The experimental points presented in Figure 9 needed to be recalculated from the original work as the data were presented in terms of mass fraction rather than volume fraction. Thus, a potential drawback of Iritani *et al*.'s data is that the solids concentration by volume is not constant across the solids composition range. However, this may not be significant for the purpose of testing the model as it does not contain, nor require, any explicit information regarding the initial suspension concentration. Instead, as is the case with the Kozeny-Carman equation, information regarding solids concentration is implicitly accounted for via the average cake porosity and solids specific surface.

From Figures 7-9 it is evident that the model reasonably represents a wide range of binary suspension filtration data using just two fitting parameters. The aforementioned data encompass filtrations of fibre/rutile mixtures at various pressures and with different solution environments, as well as filtrations of silica/rutile mixtures at two different pH values (one induced aggregation and the other did not).

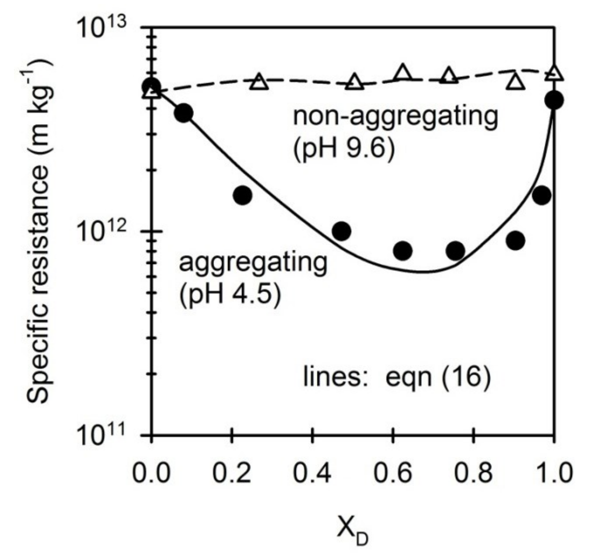

Figure 9: Experimental data for 196 kPa filtrations of silica/rutile mixtures in DI water at pH 4.5 (aggregation occurs) and pH 9.6 (no apparent aggregation), and model fits using equation (16). Here, $X_D = 0$ refers to pure rutile and $X_D = 1$ refers to pure silica. The experimental points were recalculated from Iritani *et al.* (2002).

For fibre/rutile filtrations, the parameters b_1 and b_2 were plotted against filtration pressure. From Figure 10 it is seen that b_1 values were generally larger than b_2 values indicating that the effects on filtration performance of adding rutile to a fibre rich suspension are more significant than adding fibres to a rutile rich suspension. The negative values of b_2 perhaps indicate

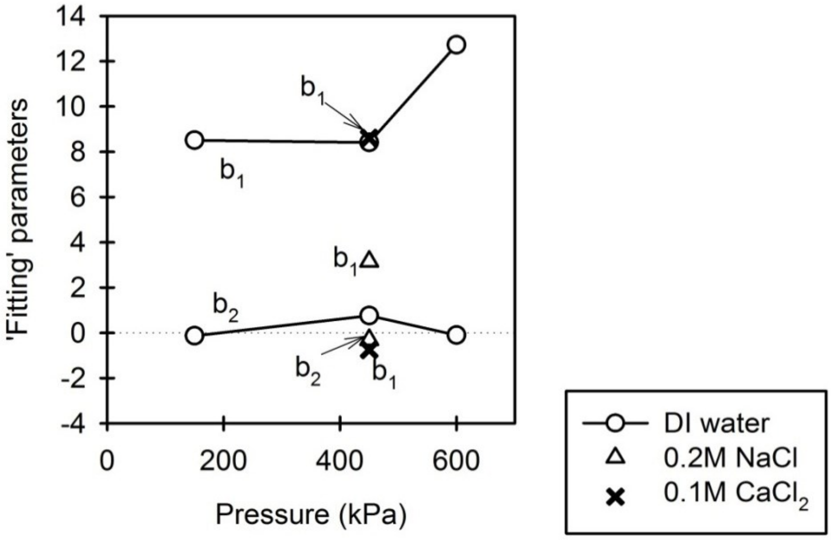

Figure 10: Values of the fitting parameters in equation (16) (b_1 and b_2) for fibre/rutile binary suspension filtrations with DI water at three filtration pressures, as well as with 0.2 M NaCl and 0.1 M $CaCl_2$ solutions at 450 kPa.

that the presence of fibres increases the rutile effective specific surface for rutile rich cakes. This increase may be possible via changes in rutile packing brought about by the presence of the fibres. For instance, the rutile primary particles could preferentially deposit onto fibre surfaces instead of forming collections of rutile clusters. Alternatively, since the negative values of b_2 are small, it may just be an experimental and/or fitting artefact with b_2 actually being ~0. Regarding the data of Iritani *et al.* (2002) for silica/rutile mixtures, with filtrations at pH 4.5 (aggregation occurred) the values of b_1 and b_2 were approximately 2.90 and 0.67, respectively. With filtrations at pH 9.6 (no aggregation), the values of b_1 and b_2 were approximately 0.48 and 0.05, respectively. Here, the values of b_1 and b_2 are more comparable, particularly when no rutile-silica aggregation was reported (pH 9.6). A likely reason is that rutile and silica are more similar in terms of their physical dimensions (size distribution and shape) compared to rutile and fibre. It could be that b_1 and b_2 are material specific parameters whose values are only valid for given solids in a given solution environment.

It should be noted that the reason for the exponential form in equation (15) was because it fits the data and used in the absence of a more fundamental model. However, considering that equation (16) appears to agree reasonably well with a range of experimental data, the possibility that there may be some underlying fundamental meaning should not be discarded. More rigorous investigation of this possibility, and indeed other approaches to modelling the experimental data which may hold some fundamental basis and/or be of practical use, is beyond the scope of this paper but would be an interesting subject for future work.

CONCLUSIONS

It was shown that, up to a given extent, interaction (aggregation) between the two solids components will strongly affect a binary cake's average specific resistance, with the magnitude of the effects varying with solids composition. The functional dependence of α_{av} on ε_{av} and the solids specific surface has been shown for binary mixtures, and the effects of aggregation on these parameters highlighted. The ratio of the specific surfaces of the two constituent solids was shown to be a critical parameter in the analysis of aggregating binary suspension filtrations. Difficulties in developing a purely theoretical model to represent the specific resistance trend with solids composition arose due to varying packing mechanisms and aggregation. A semi-empirical model was proposed which represented a wide range of experimental data reasonably well.

The fitting parameters in the semi-empirical model (b_1 and b_2) may be practically useful for characterisation purposes and/or to increase fundamental understanding of binary suspension filtration. For example, the understanding of how these parameters vary with solids form and the effects of solution environment and filtration pressure on b_1 and b_2 could be informative and insightful.

NOMENCLATURE

b_1, b_2	fitting parameters defined by equation (15) (-)
L	cake thickness (m)
ΔP	filtration pressure (Pa)
S_B	ratio of the larger component's specific surface to the smaller component's specific surface in a

	binary mixture (-)
S_m	total effective specific surface of solids in a binary mixture ($m^3\ m^{-2}$)
S_i/φ_i	specific surface of the larger (i = 1) or smaller (i = 2) solids component in contact with the permeating fluid ($m^3\ m^{-2}$)
u	superficial filtrate velocity ($m^3\ m^{-2}\ s^{-1}$)
v	true volume (excluding voids) of cake per unit area ($m^3\ m^{-2}$)
w	mass of solids per unit area ($kg\ m^{-2}$)
X_D	volume ratio of the larger solids component to the total solids in a binary mixture (-)
α_{av}	average specific cake resistance ($m\ kg^{-1}$)
μ	liquid viscosity (Pa s)
ε_{av}	average cake porosity (-)
ρ_s	solids density ($kg\ m^{-3}$)

Subscripts

s	referring to solids phase
1	referring to the (larger) solids component 1 (fibres in this work)
2	referring to the (smaller) solids component 2 (rutile in this work)

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